

Coexisting Pulses in a Model for Binary-Mixture Convection

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Abstract

We address the striking coexistence of localized waves ('pulses') of different lengths which was observed in recent experiments and full numerical simulations of binary-mixture convection. Using a set of extended Ginzburg-Landau equations, we show that this multiplicity finds a natural explanation in terms of the competition of two distinct, physical localization mechanisms; one arises from dispersion and the other from a concentration mode. This competition is absent in the standard Ginzburg-Landau equation. It may also be relevant in other waves coupled to a large-scale field.

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The rich dynamics of convection in binary mixtures has been studied extensively in recent years (e.g. [1]). This system has served as an important paradigm of extended dynamical systems which support nonlinear dissipative waves. Perhaps the most striking phenomenon observed is the localization of traveling waves into ‘pulses’; in certain parameter regimes convection does not fill the entire experimental cell but only a small part of it [2–6]. These pulses have raised various puzzling theoretical questions regarding their localization mechanism and their propagation velocity.

Most recently, experimental investigations [7] and numerical simulations of the full Navier-Stokes equations [8] have revealed in addition a surprising multiplicity of pulses of different lengths. This coexistence of different pulses at the same experimental parameters has no natural explanation within the standard complex Ginzburg-Landau equations which are commonly used for the description of weakly nonlinear waves. This draws into question the previously established theoretical understanding of the localization mechanism of the waves, which is based on their dispersive behavior. In this Letter we show that a natural understanding of the experimental as well as the numerical results is provided by an extension of the Ginzburg-Landau equations which was introduced previously to explain the anomalously slow drift of the pulses [9] and which captures an additional localization mechanism [10].

The essential features of the experiments and the numerical simulations can be summarized as follows. In the experiments [7] pulses have been investigated for three values of the separation ratio Ψ , which measures the alcohol concentration of the mixture. Their length varied from $L = 5$ to $L = 20$ as measured in vertical gap widths. For $\Psi = -0.167$ and $\Psi = -0.21$ the pulse length was found to be unique for any given Rayleigh number and all observed pulses were stable. For $\Psi = -0.127$, however, the pulse length was found to be not unique over a range of Rayleigh numbers, and depending on initial conditions pulses of two different lengths were observed. While the shorter pulse was stable, the longer pulse was unstable and could only be found by employing a suitable servo control. Strikingly, the stability of long pulses seems to be related to their drift velocity: the *stable*, long pulses

($\Psi = -0.167$ and $\Psi = -0.21$) drift *backward*, i.e. the drift velocity is opposite to the phase velocity of the waves inside the pulse, while the *unstable*, long pulses drift *forward* ($\Psi = -0.127$).

In the numerical work the full Navier-Stokes equations were simulated in a quasi-two-dimensional geometry [11,8]. Stable pulses of a shape similar to that of the experimental ones were found. As in the experiment, multiple pulse solutions were found to coexist for the same Rayleigh number (for $\Psi = -0.25$). In contrast to the pulses observed in experiments, however, all numerical pulses travel forward (for $\Psi = -0.08$ as well as $\Psi = 0.25$) and *both* coexisting pulses were found to be *stable*.

Previous analytical investigations aimed at explaining the pulses have centered around the complex Ginzburg-Landau equation (CGL). Investigation of two different limits of the CGL showed that dispersion can provide a localization mechanism for pulses; while for strong dispersion (i.e. large imaginary parts of the coefficients) the pulses correspond to perturbed soliton solutions of the nonlinear Schrödinger equation [12,13] they can be considered as a bound pair of fronts for weak dispersion [14,15]. In either case up to two solutions exist for given parameters, one stable and one unstable. However, it is always the *large* pulse which is *stable* and the *small* one which is *unstable*. A similar result was found in the CGL with additional gradient terms [16]. The unstable solution constitutes the ‘critical droplet’ which separates the stable pulse solution from the conductive state. Thus, neither the long, unstable pulses observed experimentally nor the coexistence of stable pulses of different lengths found in the numerical simulations can be understood within the CGL. In addition, within the CGL the stability of pulses is independent of the propagation direction since the group velocity can be scaled away.

Here we show that the experimental and the numerical results can be understood quite naturally within the same set of extended Ginzburg-Landau equations (ECGL) that was introduced previously in the context of the anomalously slow drift of the traveling-wave pulses [9]. We show that the multiplicity and stability of the experimental and the numerical pulses can be seen to arise from the competition of two different localization mechanisms:

one due to dispersion as in the CGL and one due to an additional concentration mode specific to the ECGL [10].

The extended Ginzburg-Landau equations describe the evolution of the complex convective amplitude A coupled to a real concentration mode C , which was introduced to describe the slow mass diffusion in liquids [9]. The relevance of such a large-scale concentration mode had been identified in the full numerical calculations of the Navier-Stokes equations [11,8]. In this paper we focus on a minimal model obtained from the equations derived in [9,17] which contains dispersion and the effect of the concentration mode on the local growth rate of the convective mode,

$$\partial_T A + s\partial_X A = d\partial_X^2 A + (a + fC)A + cA|A|^2 + p|A|^4 A, \quad (1)$$

$$\partial_T C = \delta\partial_X^2 C - \alpha C + h_2\partial_X |A|^2. \quad (2)$$

The coefficients in (1), except for the group velocity, are in general complex, whereas those in (2) are real; they are functions of the system parameters such as the Lewis number, which characterizes the ratio of mass to heat diffusion, and the separation ratio Ψ , and are given explicitly in [17] for the case of free-slip-permeable boundary conditions. Since it is known that the top and bottom boundary conditions have a strong effect on the values of the coefficients we take the values given in [17] only as an indication of the behavior of the coefficients with realistic boundary conditions. Thus, we take $h_2 f_r$ to be positive ($f \equiv f_r + if_i$), i.e. the advection of alcohol by the traveling convection rolls lead to a reduction of the local buoyancy of the fluid ahead of the pulse [17]. Such a large-scale current has been identified in the full numerical simulations of [8]. In addition, we follow [17] and assume $h_2 f_r$ to increase with decreasing Ψ . Physically, this can be understood by the fact that the phase velocity of the waves increases with decreasing Ψ which enhances the large-scale current. For simplicity, we neglect the effect of the concentration mode on the local frequency of the wave, $f_i = 0$. In the framework of a perturbed nonlinear Schrödinger equation it has been discussed in [18].

In view of the experimental and the numerical results [7,8] we solve (1,2) numerically and

study, in particular, the dependence of the length of pulse solutions of (1,2) on a_r , which is proportional to the Rayleigh number. Motivated by the dependence of the coupling strength $h_2 f_r$ on Ψ we focus first on two cases: a case of strong coupling to the concentration mode and a case of weaker coupling.

Fig.1 gives the pulse length L as a function of a_r for two values of the coupling h_2 ; $h_2 = 0.3$ and $h_2 = 0.05$ (inset). The remaining parameters are indicated in the caption. For $h_2 = 0.3$ there is a branch of solutions (solid circles) on which the pulse length increases monotonically with a_r and the pulses are stable all along the branch. The branch of unstable solutions (open circles) separates the stable pulse solution from the conductive state. These solutions constitute therefore the critical droplet. Due to the strong reduction of the buoyancy ahead of the pulse the stable and the unstable pulses travel backward.

With decreasing coupling the pulse velocity increases and for $h_2 = 0.05$ (inset) the pulses travel forward for all a_r . As in the case $h = 0.3$, there is a branch corresponding to the very short, unstable critical droplets (open circles) and a branch of short, stable pulses (solid circles). Now, however, the latter branch turns around for larger distances rendering the long pulses unstable.

The simplicity of the ECGL allows the identification of the mechanisms that lead to the stability results shown in fig.1 by considering the pulses as consisting of two interacting fronts¹. The interaction of such fronts has been studied previously in two different limits. The contribution from dispersion alone has been investigated within the CGL in the limit of widely separated fronts (L large) [15,14]. The interaction *via* the concentration mode alone has been studied within the ECGL in the limit of fronts with narrow width ξ [10]. Based

¹In the present paper we are not addressing the interaction between pulses. As shown in [19], the central features of the experimental results on the interaction [20] can also be understood within (1,2). Striking differences are found as compared to the interaction without the concentration mode [21].

on these two different analyses we expect that the essential features of the combination of both effects can be modelled by an evolution equation for the length L of the pulse which has the form² (see also [10])

$$\partial_T L = k_0(a_r - a_r^e) - k_1 e^{-L/\xi} + \frac{k_2}{L} - \frac{k_3}{v} e^{-\alpha L/|v|}. \quad (3)$$

The first term describes the invasion of the conductive state by the convective state for a_r larger than the equilibrium value a_r^e ($k_0 > 0$). The second term represents the attractive interaction ($k_1 > 0$) between fronts which arises already within the real Ginzburg-Landau equation. The third term gives the contribution to the interaction from dispersion. In the absence of the concentration mode, stable pulses exist only if $k_2 > 0$. The last term captures the interaction *via* the concentration mode C . It decays over a length which depends on the damping α of C and the velocity v of the pulse. In the limit considered in [10] $k_3 = \sqrt{12}h_2 f_r$. For the regime in question k_3 is therefore positive [17,8]. Most importantly, the sign of the interaction depends on the direction of propagation of the pulse.

To interpret the numerical results shown in fig.1 we plot in fig.2 the right-hand-side of (3) for $v < 0$ (inset) and for $v > 0$. For $v < 0$ (3) allows three solutions. The longest pulse is stable (full circle) and the shorter one unstable (open circle). The smallest pulse (open diamond) has $L = O(\xi)$ and falls outside the range of validity of (3). This corresponds to the strong-coupling case $h_2 = 0.3$ in fig.1. For forward traveling pulses, $v > 0$, (3) has five solutions: two stable, two unstable, and one unphysical solution. This corresponds to the weak-coupling case ($h_2 = 0.05$) if the stable solution in fig.1 is identified with pulse B and the unstable ones with A and C. Note, that increasing a_r raises the curve in fig.2 and leads to a merging of solutions B and C as in fig.1. For these parameter values no stable long pulses corresponding to solution D were found in the numerical simulations of (3) (see below).

²More precisely, one would expect coupled equations for the velocities of the leading and of the trailing front which both depend also on the distance between them [10].

The results shown in fig.1 agree qualitatively very well with the experiments of Kolodner (cf. figs.32,33 in [7]). There also, short pulses are stable for all Ψ investigated, independent of their direction of propagation. Long pulses, on the other hand, are only stable for strongly negative Ψ (i.e. strong coupling) for which the pulses travel backward. For positive velocity (larger Ψ) the long pulses are unstable. Most importantly, the comparison of fig.1 and fig.2 allows the identification of the relevant mechanisms in the different experimental regimes. For short pulses the dispersive interaction dominates and the localization mechanism is essentially as given by the CGL. For the longer pulses, however, the interaction *via* the concentration mode is stronger. Therefore, the pulse stability is closely related to the propagation velocity and long pulses are unstable for weakly negative Ψ . The cross-over between the two mechanisms can be seen in the experimental data for strongly negative Ψ where the Rayleigh-number dependence of the length shows a clear break at intermediate lengths.

As mentioned earlier, in the numerical simulations of the Navier-Stokes equations multiple pulse solutions have been obtained as well [8]. In contrast to the experiments, however, the short as well as the long pulse were found to be stable. This difference may be due to the fact that in the numerical simulations a quasi-two-dimensional geometry was employed while the experiments were done in a narrow channel³.

Are the ECGL (1,2) able to account also for this coexistence of *stable* pulses of different length? The evolution equation (3), which is conjectured based on (1,2), suggests that quite generally there should be a second stable pulse (the long pulse D) since the power-law behavior of the dispersive term dominates the exponentially decaying interaction *via* the concentration mode for very large L . A more detailed analysis of the dispersive interaction shows, however, that the power-law behavior persists only up to some maximal length L_{max} which decreases with increasing dispersion [15]. To study the dispersive interaction over

³The difference in geometry will also be reflected in the values of the coefficients of (1,2).

the full range of L we investigate the ECGL in the absence of the concentration mode (i.e. the usual CGL). Fig.3 shows a_r as a function of the inverse length of the pulse, $1/L$, for various different values of the dispersion c_i . Clearly the power-law behavior is only found up to some maximal length L_{max} which decreases with increasing dispersion. Beyond this length the dispersive interaction decays extremely rapidly. Therefore, if in the full ECGL the concentration mode should dominate for lengths up to L_{max} and beyond, dispersion will not stabilize long pulses.

Thus, the coexistence of short as well as long stable pulses obtained in [8] is only expected to arise if the contribution from the concentration mode decays sufficiently fast. Fig.4 shows the results of simulations of (1,2) with increased damping α as well as larger coupling strength f_r . Indeed, two unstable and two stable pulses are found, in agreement with the evolution equation (3). This gives a natural explanation of the coexistence of stable pulses obtained in the full numerical simulations [8] in terms of an alternating dominance of two different localization mechanisms.

The robustness of the localization mechanism due to the concentration mode suggests that the extended Ginzburg-Landau equations may also allow localized chaotic pulses. Temporally chaotic localized waves have been found previously in the complex Ginzburg-Landau equation [22] as well as in the extended Ginzburg-Landau equation in the absence of dispersion [23]. In these cases it is essentially the length of the pulse which varies chaotically; the traveling wave state within the pulse is regular. In the presence of dispersion the extended Ginzburg-Landau equation possibly allows waves which are localized by the concentration mode and which at the same time are Benjamin-Feir unstable and exhibit phase chaos.

In conclusion, we have shown that the extended Ginzburg-Landau equation can account for the essential features of the localized waves found in experimental and numerical investigations of binary-mixture convection; the ordering of stable and unstable pulses as well as the coexistence of stable pulses of different lengths is seen to be a natural consequence of the competition between the interaction between fronts due to dispersion and due to the concentration mode. The latter depends sensitively on the direction of propagation of the

pulse. Thus concentration-dominated long pulses are stable only if they travel backward in agreement with the experimental results [7]. If the concentration mode decays sufficiently fast in space two coexisting stable pulses of different lengths can arise as found in numerical simulations of the Navier-Stokes equations [8]. The results demonstrate that the concentration field plays a crucial role in the stability and the dynamics of the pulses. Its essential features are captured at least qualitatively by the extension of the Ginzburg-Landau equations discussed in this paper. It is worth mentioning that the extended Ginzburg-Landau equations apply also to interfacial waves in Poiseuille flow [24] and waves coupled to a large-scale field in general. They may also be relevant for electroconvection in nematic liquid crystals [25].

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FIGURES

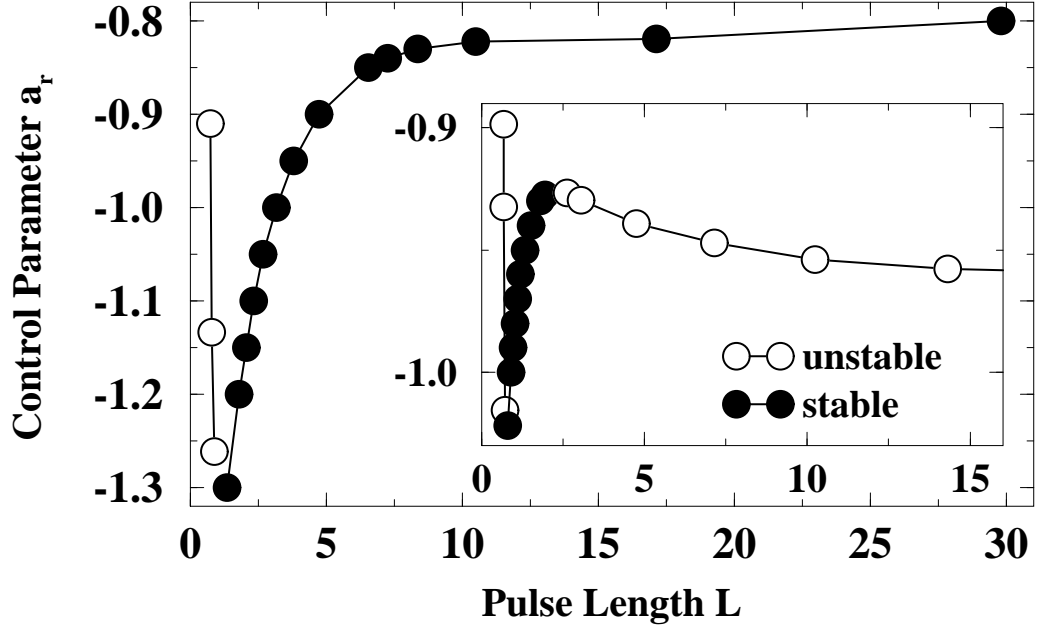


FIG. 1. Pulse length *vs.* growth rate a_r for $d = 0.01$, $c = 2.45 - 0.5i$, $p = -1$, $\delta = 0.009$, $\alpha = 0.02$, $s = 0.1$ and $f = 0.1$. For $h_2 = 0.3$ the pulses travel backward for all values of a_r whereas for $h_2 = 0.05$ (inset) they travel forward.

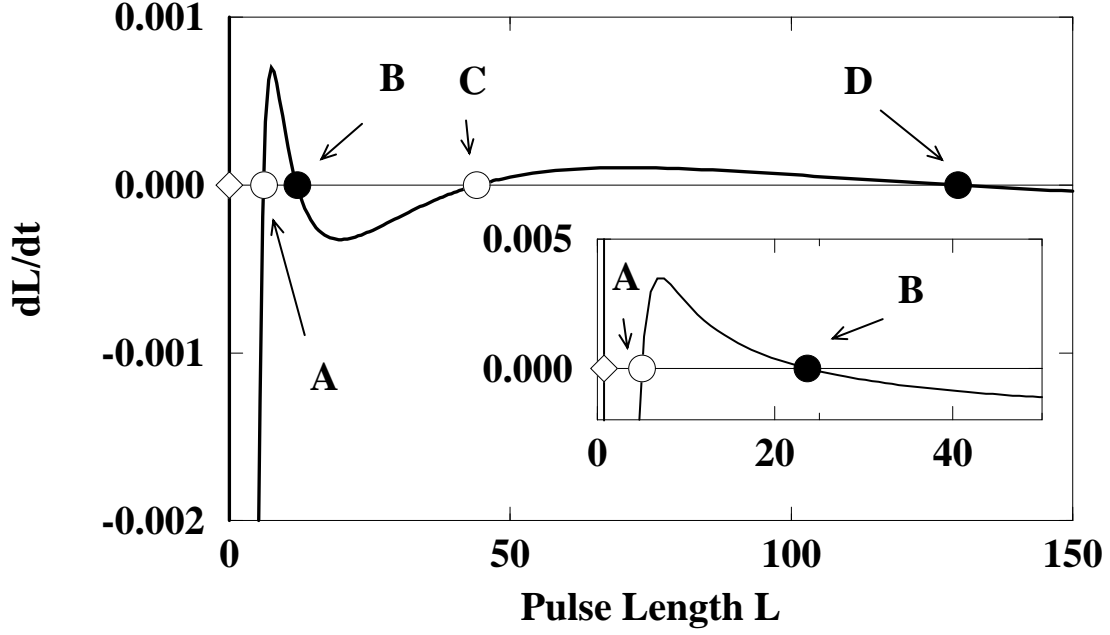
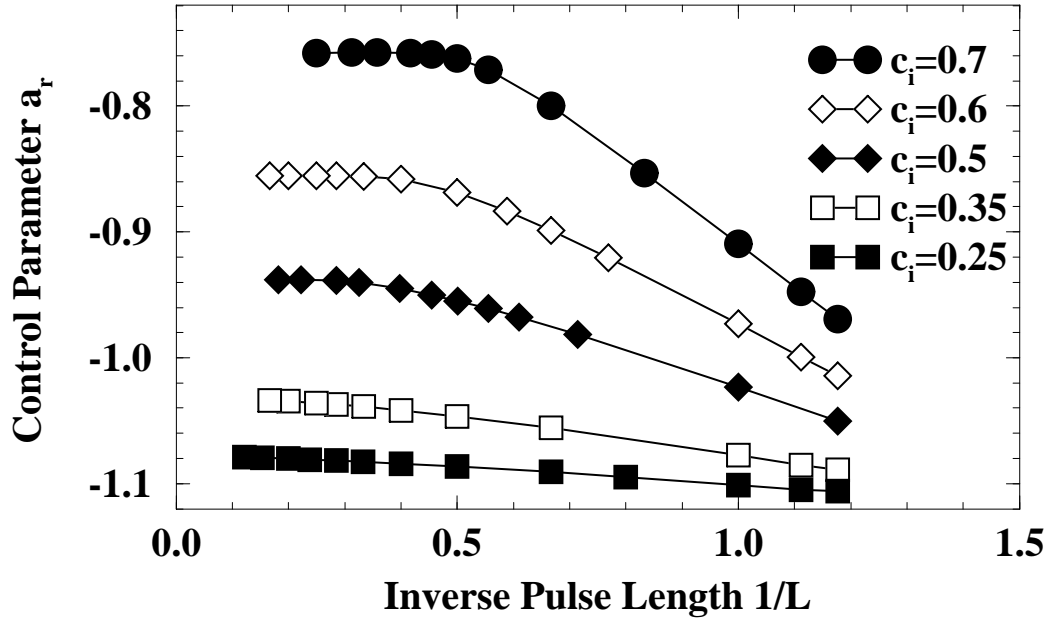


FIG. 2. Steady-state solutions of (3): $dL/dt = -0.0003 - e^{-x} + 0.04/x - 0.0055e^{-0.05x}$. The solid circles indicate stable pulse solutions the open circles unstable ones. The diamond gives the unphysical solution with $L = O(\xi) \equiv O(1)$. The inset shows steady-state solutions of (3) for $v < 0$: $dL/dt = -0.002 - e^{-x} + 0.04/x + 0.001e^{-0.05x}$.



Fig

FIG. 3. Pulse length L *vs.* control parameter a_r in the absence of the concentration mode ($f = 0$) for $d = 0.01$, $c_r = 2.45$, $p = -1$. The power-law behavior extends only up to a maximal length L_{max} which decreases with increasing dispersion.

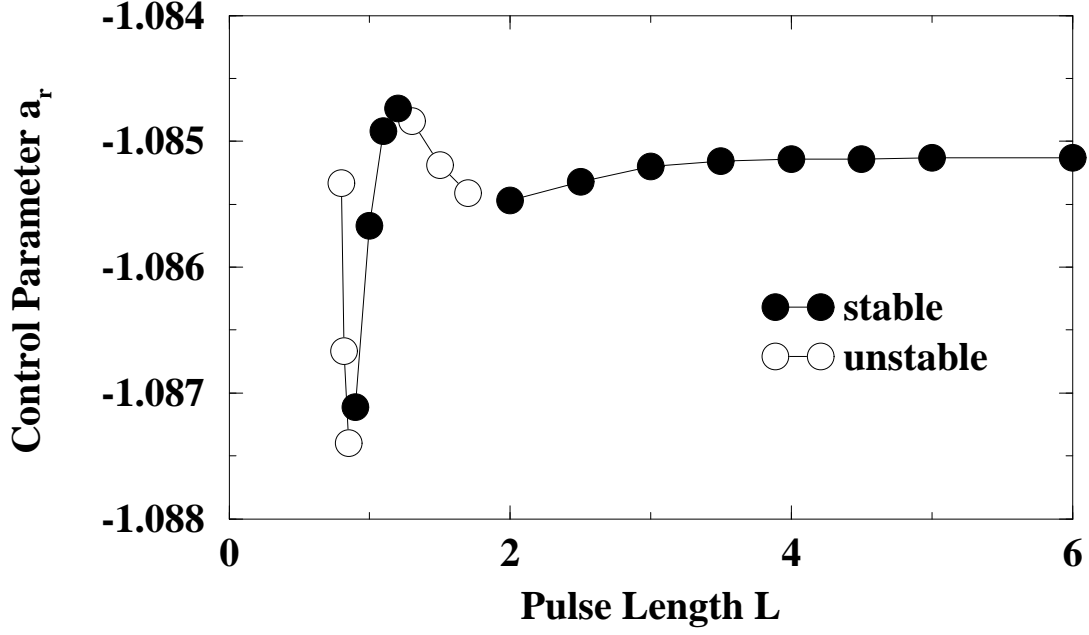


FIG. 4. Pulse length *vs.* growth rate a_r for a rapidly decaying concentration mode ($d = 0.01$, $c = 2.45 - 0.25i$, $p = -1$, $\delta = 0.03$, $\alpha = 0.5$, $s = 0.1$, $h_2 = 0.05$ and $f = 0.45$). For large L the dispersion becomes dominant again and restabilizes the pulses leading to the coexistence of stable pulses.